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Regularities in the Response of Spectral Lines to Small Perturbations in Physical Quantities in the Photosphere

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Abstract—Numerical simulations are used to establish a number of dependencies between small perturbations in physical quantities in the photosphere and small variations in the Stokes profiles of spectral lines. A perturbation of any physical quantity in the model photosphere shifts every point in a line profile in the direction perpendicular to the tangent to the profile at that point. The actions on the wing of a spectral line of perturbations in the magnetic field and radial velocity are equivalent for a particular ratio of these perturbations (if the line is fully split in the magnetic field). If the response of part of a line wing is considered as a shift in wavelength, the area under the curve representing the response to perturbations in the magnetic field and radial velocity has a simple physical meaning.

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1. INTRODUCTION

The interpretation of observations of Stokes profiles of spectral lines is one of the main sources of information about the structure of the photosphere and active formations in it. This interpretation is done using some kind of inversion method. Most often this inversion is based on a Milne–Eddington model [1, 2], or Stokes Inversion based on Response functions [3]. Inversion procedures establish the relationship between Stokes profiles and the values of physical quantitites at specified heights in the photosphere. Inversion procedures operate automatically, and do not provide the researcher a clear picture of how exactly a Stokes profile is related to various physical quantities in the photosphere. We have carried out a series of model computations in order to uncover these relationships, in which small variations in physical quantities were added to photospheric layers in order to trace small variations in the profiles. This has made it possible to draw a number of generalizations and obtain a number of simple relations for which we could not find references in the literature. These relations could be used to make a priori estimates before carrying out an inversion procedure, reducing the number of computations and enhancing the quality of the results.

2. COMPUTATIONAL METHOD

This study is based on the following numerical method for calculation of the response functions. We first computed the profile of a spectral line for a specified model photosphere, which served as a reference profile. The model photosphere was then changed at a specified height h in a narrow "test" layer of width Δh: a selected physical quantity X was increased by a test amount ΔX. The spectral-line profile was again computed, and the difference from the reference profile ΔY was calculated, where Y is some characteristic of the profile. By moving the test layer through all heights in the photosphere h and measuring the differences between the perturbed and reference profiles, we constructed the Response Function (RF) $\Delta Y(h)$ to variations in the physical quantity X. This method was first applied by Wittman [4]. The width of the test layer Δh and the magnitude of the test perturbation ΔX must be fairly small. Otherwise, the test layer distorts the model, and the response function will no longer correspond to the studied photospheric model. To avoid distortion of the shape of the RF, we chose a small width of the test layer (5 km) and small growths in the physical quantities in the test layer.

The computation of the spectral-line profiles was done via numerical integration of the equations of radiative transfer in the Stokes parameters for a one-dimensional model photosphere. Scattering in the line and deviations from LTE were not included.

3. RELATION BETWEEN SMALL VARIATIONS OF A SPECTRAL-LINE PROFILE IN INTENSITY AND WAVELENGTH

Small variations in the radial velocity lead to a shift of a spectral-line profile in wavelength. On the

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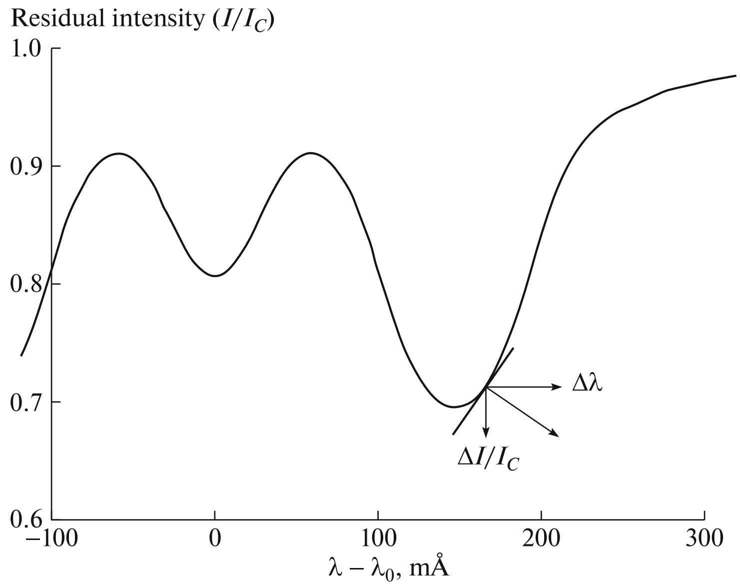

Fig. 1. Illustration of how small variations of any of the physical quantities in any layer of the atmosphere lead to a shift of each point in the profile in the direction perpendicular to the tangent at the given point. This means that, for any point in the profile, it is always possible to find $\Delta\lambda$, knowing ΔI, and vice versa.

other hand, small variations in the temperature lead primarily to variations in intensity. Following this reasoning, we asked ourselves whether it is possible to separate information about the influence on a profile due to variations in the radial velocity and temperature, considering separately the profile variations in wavelength and residual intensity. Our computations showed that, at any point of a line profile and at any height for the perturbed layer, the variations in the intensity profile as a function of wavelength λ and as a function of residual intensity I/I_C are always related by the expression

$$\Delta I = \Delta\lambda(dI/d\lambda), \quad (1)$$

where $dI/d\lambda$ describes the slope of the wing at a given point in the profile. We verified the validity of this relation by considering variations in the magnetic field B, temperature T, radial velocity v_{LOS}, and microturbulence velocity v_{mic}. Thus, a small perturbation of any physical quantity in the model photosphere shifts each segment of a line profile in the direction perpendicular to this segment (see Fig. 1).

The relations for the other Stokes parameters (Q, U, V) are analogous: $\Delta Q = \Delta\lambda(dQ/d\lambda)$, and so forth. This result leads to the following conclusions.

1. Comparing two experimental profiles, we can analyze either the difference ΔI or the difference $\Delta\lambda$, depending on what is more convenient or expendient in a specific situation.

2. Small variations at a single point in a line profile due to variations in the temperature and other physical quantities cannot be distinguished.

4. EQUIVALENT ACTION ON A SPECTRAL-LINE WING BY PERTURBATIONS OF THE RADIAL VELOCITY AND MAGNETIC FIELD

The dependence of responses of a line profile to small variations in some physical quantity as a function of the height h where the variations are introduced is the response function RF. Variations in the radial (line-of-sight, LOS) velocity Δv_{LOS} and of the magnetic field ΔB in a height interval Δh in the photosphere shift a section of the wing of the profile of a magnetoactive spectral line in wavelength an amount that is proportional to Δv_{LOS} or ΔB. We expect that the RFs for these physical quantities will be very similar. Indeed, our computations showed that, for a given ratio $\Delta v_{\mathrm{LOS}}/\Delta B$, the RFs are almost identical both in magnitude and as a function of h (see Fig. 2).

The reduction in RF_B relative to RF_v_{LOS} in the line wing ($I/I_C = 98\%$) in Fig. 2 is due to the fact that the absorption profile becomes appreciably broader here, so that the opposite σ components of the Zeeman splitting overlap and partially cancel each other. If the computations are repeated for small magnetic fields, when the mutual blending of the σ components is strong, RF_B and RF_v_{LOS} will obviously differ.

We will consider what ratio $\Delta v_{\mathrm{LOS}}/\Delta B$ is required for RF_B and RF_v_{LOS} to be the identical in the following sections.

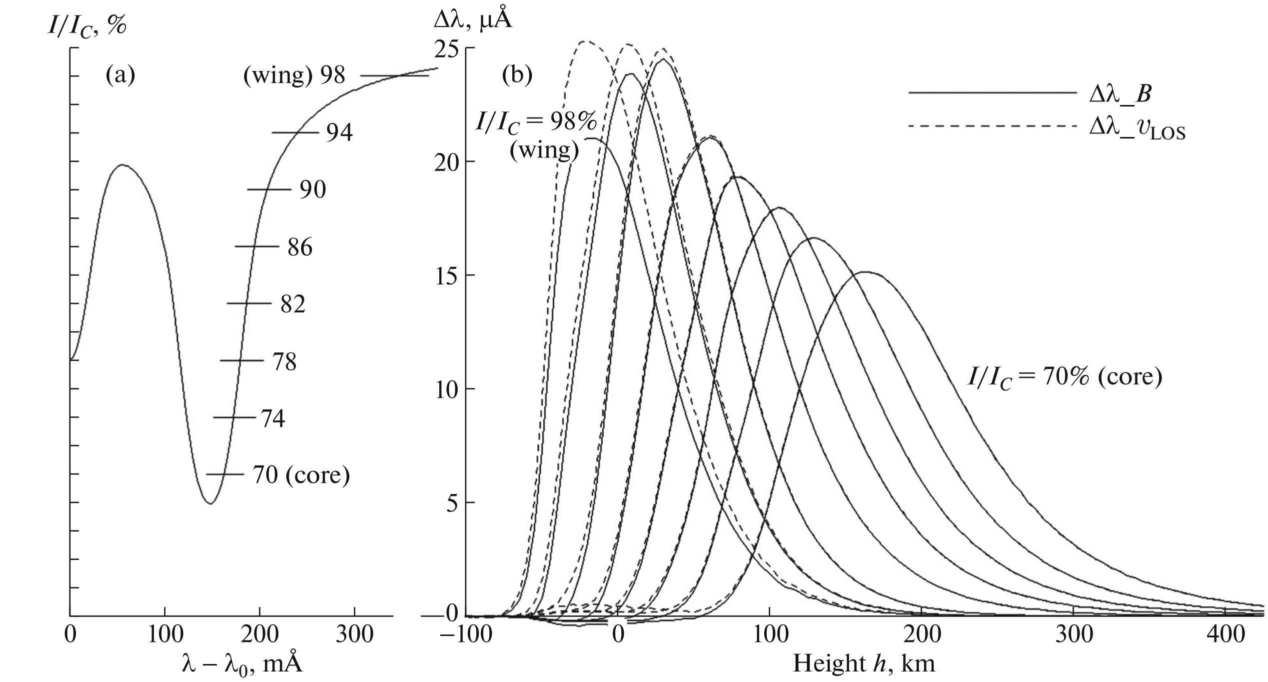

Fig. 2. Coincidence of the response functions RF_B and RF_$v_{\rm LOS}$ at a certain ratio of the test perturbations $\Delta v_{\rm LOS}/\Delta B$. RFs for individual points of a profile that correspond to the specified residual intensities I/I_C (with a step of 4%) were considered. The (a) positions of the corresponding points in the intensity profile and (b) the wavelength shift for the response ($\Delta\lambda\_B$ and $\Delta\lambda\_v_{\rm LOS}$) as a function of the height of the perturbed layer (h) are shown. The values of the RFs are related to the responses as $\mathrm{RF} = \Delta\lambda/(\Delta h \cdot \Delta X)$, where ΔX is $\Delta v_{\rm LOS}$ or ΔB. The computations were carried out using a Stellmacher–Wiehr model photosphere [5], with a microturbulence velocity $v_{\rm mi} = 0$, magnetic field strength $B = 3200$ G, and inclination of the field to the line of sight $\gamma = 15°$, for the Fe I λ 6302.5 Å. The width of the test layer was $\Delta h = 5$ km, and the physical quantities variations were $\Delta B = 10$ G, $\Delta v_{\rm LOS} = 21.94$ m s^{-1}.

5. PHYSICAL MEANING OF THE AREA UNDER THE RF CURVE FOR PERTURBATIONS OF THE RADIAL VELOCITY AND MAGNETIC FIELD

The plots in Fig. 2 were constructed by joining the peaks of bins of a histogram. The width of the bins $\Delta h = 5$ km corresponds to the width of the test layer. The height of a bin is the wavelength shift $\Delta\lambda_h$ due to the variations $\Delta v_{\rm LOS}$ or ΔB. We calculated the sum over all h values of the wavelength shifts, $\Delta\lambda_\Sigma = \sum_{\forall h} \Delta\lambda_h$, for various plots corresponding to points in a profile with different residual intensities I/I_C. This sum is a constant for all the plots for the various I/I_C values (apart from the plots for B for $I/I_C = 90$, 94, and 98%, where $\Delta\lambda_\Sigma$ successively decreases). It is clear that this constancy has not come about by chance.

The quantity $\Delta\lambda_h$ is proportional to the width of the test layer Δh and the value of the test signal ΔX. Let us suppose that this proportionality is strict, so that we can write

$$\Delta\lambda_h = \mathrm{RF}_h \cdot \Delta h \cdot \Delta X, \qquad (2)$$

where RF_h is the RF at height h. We denote the area under the RF curve $S_{\rm RF}$:

$$S_{\rm RF} = \sum_{\forall h} (\mathrm{RF}_h \cdot \Delta h) = \sum_{\forall h} (\Delta\lambda_h)/\Delta X. \qquad (3)$$

Then,

$$\sum_{\forall h} (\Delta\lambda_h) = S_{\rm RF} \cdot \Delta X \qquad (4)$$

and the quantity $S_{\rm RF}$ has a simple physical meaning. It is equal to the ratio for converting a variation in a physical quantity into a wavelength shift. In other words, the sum of the shifts $\sum_{\forall h} (\Delta\lambda_h)$ obtained by moving the test layer through all heights is equal to the shift acquired by a point in the profile if the test perturbation is applied to the entire photosphere simultaneously. For the radial velocity, the area under the RF curve is essentially equal to

$$S_{\mathrm{RF}\_v} = \lambda/c, \qquad (5)$$

and for the magnetic field, it is equal to

$$S_{\mathrm{RF}\_B} = 4.67 \times 10^{-5} \lambda^2 G_{\rm Lande}, \qquad (6)$$

where c is the speed of light, G_{Lande} the Landé factor of the spectral line, and λ its wavelength. The equalities (5) and (6) are approximate, and their left-hand sides were obtained empirically, using the results of the computational experiment for the ratios $S_{\mathrm{RF\_}v} = \Delta\lambda_\Sigma/\Delta v_{\mathrm{LOS}}$ and $S_{\mathrm{RF\_}B} = \Delta\lambda_\Sigma/\Delta B$, while the right-hand sides are calculated substituting the values for c, λ, and G_{Lande}. The deviations of (5) from the exact expression were found to be within the uncertainties of the numerical integration of the equations of radiative transfer for any initial conditions; in particular, we verified this for various model atmospheres. The equality (6) remains within the computational uncertainties until the opposite σ components of the Zeeman splitting begin to be superposed at small B values, so that they are blended with each other.

6. CONCLUSIONS

1. When comparing experimental and computed profiles, as a rule, the difference between the intensities of points with the same wavelengths are estimated. However, when analyzing radial velocities or magnetic fields, it is physically more correct to estimate the difference of the wavelengths for points with the same intensities. Our results show that these two ways of comparing profiles are mutually interchangeable.

2. It is not possible to distinguish the action of different physical quantities on a single specific point in a profile, and to distinguish the influence of temperature T, radial velocity v_{LOS}, microturbulence velocity v_{mic}, magnetic field B, etc. The action of these factors is superposed independently and linearly. It is always possible to compute the growth in T, v_{LOS}, v_{mi}, and B at a height h that give equivalent variations of a particular section of a line profile.

3. The action on a specified line wing by variations of v_{LOS} and B are equivalent for the ratio $\Delta B/\Delta v_{\mathrm{LOS}} = 4.67 \times 10^{-5}\lambda G_{\mathrm{Lande}}/c$, where c is the speed of light, G_{Lande} the Landé factor of the spectral line, and λ its wavelength. This equivalence is disrupted when the separation of the σ components in the magnetic field becomes incomplete. This equivalence is preserved independent of the height of the test layer h into which the variations of these physical quantities are introduced, and of the point in the profile wing considered (i.e., of the wavelength).

4. Separating information about various physical parameters in the photosphere requires the consideration of different sections of spectral-line profiles (or multiple lines). It is obvious that separation of information about the radial velocity and magnetic field requires a comparison of oppositely placed sections of two wings of the same spectral line, preferably in a region with the maximum values of $dI/d\lambda$. Extraction of information about the temperature using inversion techniques is best carried out using sections of profiles with minimum values of $dI/d\lambda$ ($dV/d\lambda$, $dQ/d\lambda$).

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