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Temperature dependence of Stokes profiles for model photospheres

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The profiles of magnetoactive lines as a function of the effective temperature of model photospheres is studied in a computer simulation. It is shown that most lines of neutral iron are most enhanced in models intermediate in temperature between models of the quiet photosphere and of a sunspot umbra. The I, V, and Q Stokes profiles have different temperature sensitivities; the reasons for such a difference are explained. The strongest I, V, and Q profiles correspond to models with successively higher temperatures. The profiles of circular polarization vary quite weakly over a wide temperature range, so they may not always be suitable for a separate analysis of the cool and hot components of a two-component umbra.

Studies of multicomponent, small-scale photospheric structures containing a magnetic field have recently become timely.[1,2] To interpret spectroscopic observations of such structures, one needs a good idea of how the characteristics of model photospheres affect the profiles of Fraunhofer lines.

ATLAS

A specific atlas of Stokes profiles of the Fe I 6302 Å line, calculated for a grid of values of the strength of the magnetic field and its inclination to the line of sight, is helpful in solving for the influence of models on a line. The profiles have been calculated by numerical methods under the assumption of LTE and pure absorption and with allowance for anomalous dispersion. The atlas, described in detail in Ref. 3, was intended for the analysis of observations of a bright bridge in a sunspot, so it was calculated for a number of models of the quiet and disturbed solar photosphere: HOLMU, the undisturbed photosphere, Holweger and Müller[4]; MOEMA, the penumbra, Moe and Maltby[5]; SOBB2, the light bridge, Sobotka[6]; ZWAAN, the umbra, Zwaan[7]; SW, the umbra, Stellmacher and Wiehr[8]; M4, a modification of the SW model, Kollatschny et al.[9]; and HM+0.2, the light bridge, a model obtained by a recalculation from the HOLMU temperature distribution under the assumption of hydrostatic equilibrium after increasing $\theta = 5040/T$ by 0.2 at each optical depth.

In Table I we give the effective values, corresponding to the models, of $\theta = 5040/T_{eff}$ and the contrast I_c of the continuum with respect to the quiet photosphere at 6300 Å. The temperature gradient for an entire model cannot be expressed by one number; the HOLMU and HM+0.2 models are identical in this sense, however, while the M4 model was obtained from the SW model precisely by an increase in the temperature gradient (at the depths at which $\tau_{0.5}$ varies from 0.3 to 1.0) and hence in $dS/d\tau$, where S is the source function.

An examination of the atlas (Fig. 1) immediately showed relationships that are not entirely obvious.

1. Absorption lines are often divided, in accordance with temperature sensitivity, into "umbral" lines, i.e., those that are enhanced in a sunspot umbra, and "photospheric" lines, which are, conversely, weakened in an umbra. The profiles

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TABLE I

Model	HOLMU	MOEMA	SOBB2	HM+0,2	ZWAAN	M4	SW
θ_{eff}	0,82	0,88	0,98	1,02	1,25	1,24	1,29
I_c	1,000	0,743	0,467	0,405	0,138	0,147	0,118

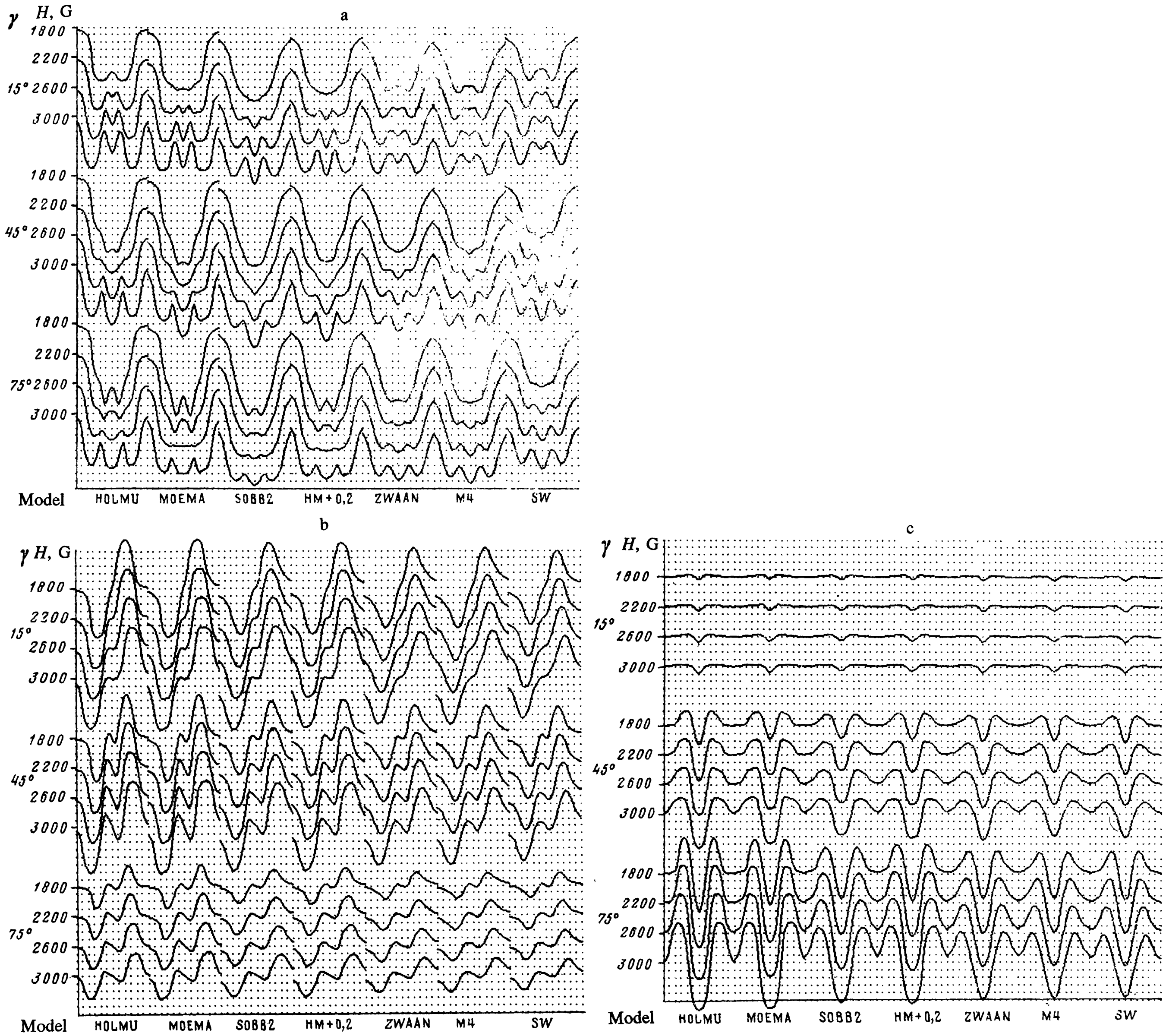

FIG. 1. Atlas of profiles of the Stokes parameters I (a), V (b), and Q (c) of the Fe I 6302 Å line, $\varepsilon_l = 3.69$ eV, for four values of the magnetic field strength H, three angles γ of inclination of field lines to the line of sight, and seven models of the quiet and disturbed photosphere. The microturbulent and macroturbulent velocities are 1.0 and 0 km/sec, respectively. The step sizes of the coordinate grid are 40 mÅ horizontally and 5% vertically.

of intensity I of the Fe I 6302 Å line turned out to be deepest for models intermediate in temperature between the models of a sunspot umbra and the undisturbed photosphere, i.e., that line is more properly called "penumbral" in accordance with its temperature sensitivity.

2. The profiles of circular (V) and linear (Q) polarization are strongest at higher temperatures than for the I profile, i.e., different Stokes parameters have different temperature sensitivities.

3. Besides the effective temperature of the model, the depth distribution of the temperature gradient, more precisely, of the quantity $dS/d\tau$, significantly affects the relative strength of the line profiles. This is seen (Fig. 1) from a comparison of intensity profiles for the M4 and SW models.

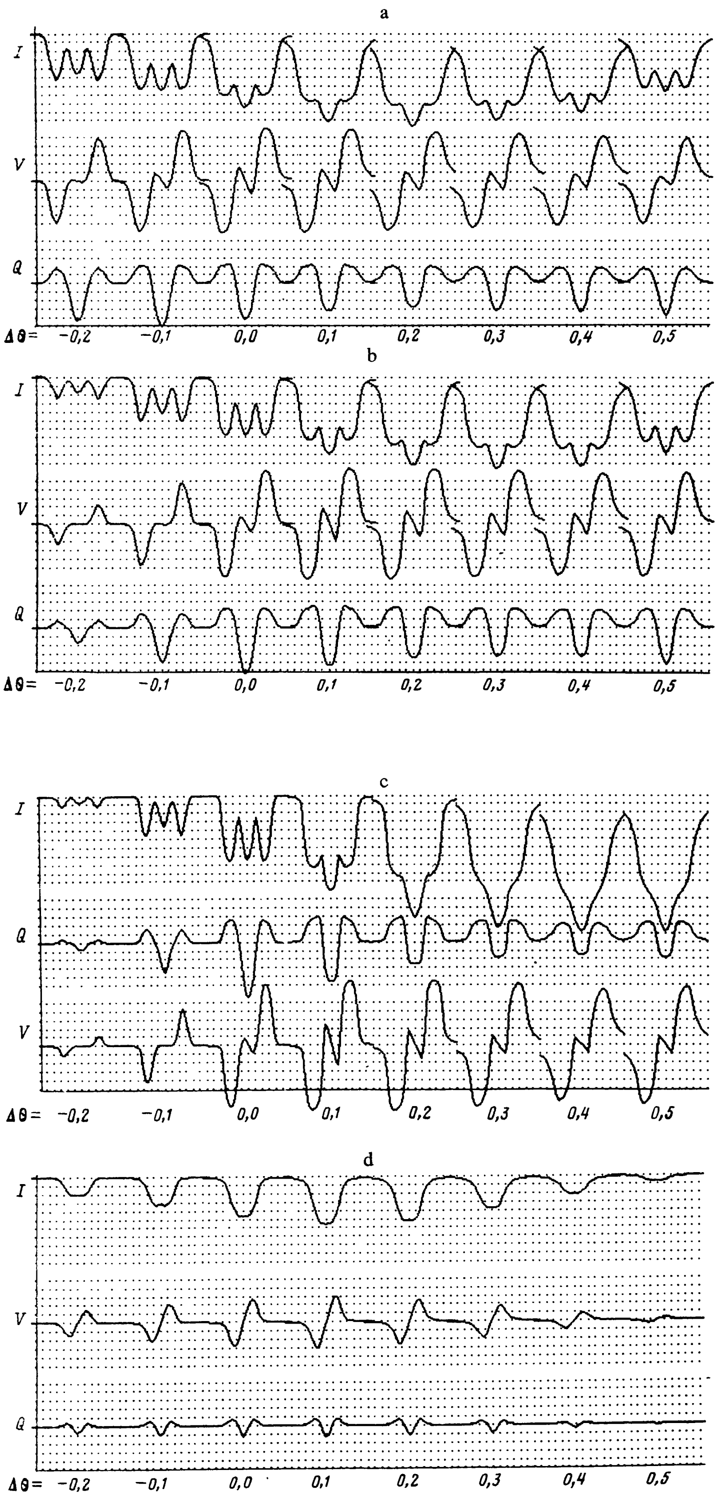

FIG. 2. Stokes profiles of the Fe I 6302 Å (a), 6173 Å (b), 5250 Å (c), and 6089 Å (d) lines for eight model photospheres obtained from the HOLMU temperature distribution. $|\mathbf{H}| = 2600$ G, $\gamma = 45°$, $V_{\mathrm{mic}} = 1.0$ km/sec, and $V_{\mathrm{mac}} = 0$. The step sizes of the coordinate grid are the same as in Fig. 1.

Those models differ only in temperature gradient, and the difference in central residual intensities is about 10%.

As additional calculations showed, the influence of depth variation of $dS/d\tau$ on the line profile is not simple. An investigation of that question is planned for a separate paper.

TEMPERATURE SENSITIVITY OF DIFFERENT LINES

A specially calculated series of models helped to eliminate the influence of variations in the temperature gradient on

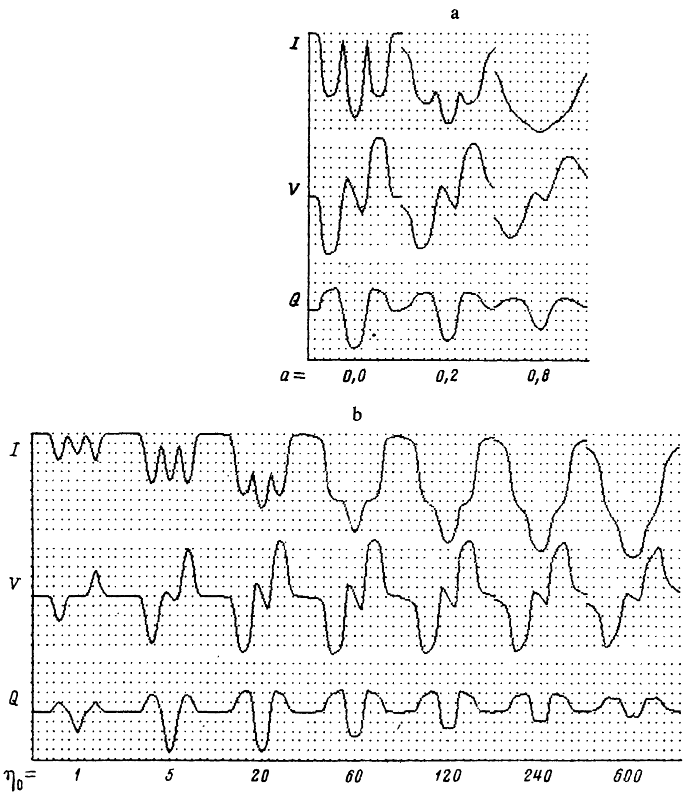

FIG. 3. Stokes profiles of a triplet line, obtained from analytic solutions of the radiative transfer equations, as functions of the following: a) the damping parameter a (η_0 = 50, v_H = 2.5 × 2); b) the ratio η_0 of the absorption coefficients at the line center and in the continuum (a = 0.02, v_H = 2.5 × 1.5). Coordinate grid same as in Fig. 1, except that $\Delta\lambda_D$ = 25 mÅ. Angle of inclination γ = 45°, Milne–Eddington parameter of the atmosphere β = 2.

the profiles. We took the HOLMU temperature distribution, and at each optical depth we added a constant amount $\Delta\theta$, the so-called model parameter, where $\theta = 5040/T$. The electron and gas pressures were then recalculated under the assumption of hydrostatic equilibrium. In such a procedure the function $dS/d\tau(\tau)$ remains unchanged (we presume the equivalence of the source function S and the Planck function B, which is admissible for LTE), and the effective value θ_{eff} changes by $\Delta\theta$. We calculated a total of eight models with values of the model parameter $\Delta\theta$ from −0.2 (θ_{eff} for a white light flare) to +0.5 (θ_{eff} for a sunspot umbra) with a step size 0.1 eV^{-1}. The Stokes profiles of different lines were then calculated for those models.

In Fig. 2 we give profiles of the Fe I 6302 Å (a), 6173 Å (b), 5250 Å (c), and 6089 Å (d) lines, the lower levels of which have excitation potentials ε_l = 3.69, 2.22, 0.12, and 5.02 eV, respectively. The 6089 Å line is a triplet with a Landé factor g = 1. It is obvious that for any spectral line a model effective temperature T_{max} must exist at which its profile is most strengthened (see Ref. 10). The quantity T_{max} depends on the element number, its ionization stage (and potential), and, as is clearly evident from Fig. 2, the excitation potential of the lower level for the given line. T_{max} may also vary slightly as a function of the oscillator strength gf of the line and specific features of the model atmosphere.

Figure 2 thus shows that Fe I lines with $\varepsilon_l \sim 0$ eV are umbral, while lines with $\varepsilon_l \gtrsim$ 1-2 eV are more properly called penumbral in accordance with their temperature sensitivity, and for $\varepsilon_l \sim 5$ eV they become photospheric.

As an aside, we note that using the 5250 Å line for magnetographic observations of facular points may lead to errors due to the strong temperature dependence of the I and V profiles for $\Delta\theta$ = −0.2-0.

DIFFERENCE IN TEMPERATURE SENSITIVITY OF THE STOKES PARAMETERS *I*, *V*, AND *Q*

Figure 2 shows more clearly than the atlas the difference in temperature sensitivity of the different Stokes parameters. First, the I, V, and Q profiles reach maximum enhancement at successively higher temperatures. Second, the Q profiles have a local minimum in temperature sensitivity where the I profiles have a maximum. Third, the V profiles vary weakly with temperature over a certain range (between the two maxima of the Q profiles).

It has been possible to explain those relationships by analyzing profiles obtained from analytic solutions of the transfer equations in the Stokes parameters for the Milne–Eddington model atmosphere. As follows from Unno's equations[11] (no change in the allowance for anomalous dispersion), the Stokes profiles are completely determined by four parameters, a, η_0, v_H, and γ, where a is the damping parameter, which determines the Lorentzian part of the Voigt profile, η_0 is the ratio of the absorption coefficients at the line center and in the adjacent continuum, which is the line strength, $v_H = \Delta\lambda_H/\Delta\lambda_D$ determines the ratio of magnetic and Doppler broadening, and γ is the angle of inclination of of magnetic flux lines to the line of sight. The parameter β in the equation specifying the Milne–Eddington model, $B = B_0(1 + \beta\tau)$, can change the scale of the profiles along the intensity axis, but does not affect their shape. The quantities a and η_0 depend strongly on the temperature of the model.

The damping parameter a increases regularly and fairly significantly with decreasing temperature. This leads to broadening of the absorption profiles of the π and σ components of a line, as a result of which their mutual overlap increases. Whereas the overlapping components are added in the intensity profile, they are subtracted in the polarization profiles. Compensation of polarization occurs in the vicinity of points at which its sign changes: it "spills over" into the intensity profile. The fact that we are dealing with the "spillover" of polarization rather than simple mutual cancellation is shown by Fig. 3a, in which we give profiles, calculated analytically from the equations of Refs. 12 and 13, of a triplet with $a = 0$, 0.2, and 0.8. The equivalent width and even the central depth of the intensity profile increase with increasing a, which seems surprising at first glance. The point is that the absorption coefficient at the line center is approximately proportional to $1 - a$, and in the absence of a magnetic field, a line actually becomes narrower with increasing a (if it has not yet reached saturation). The change in this picture upon the appearance of a field can be represented clearly as the result of "spillover" from the polarization profiles.

The effective value of the ratio of coefficients of selective and continuous absorption, $\eta_{0,eff}$, has a maximum at a certain effective temperature T_{eff} of the model. The quantity $\eta_{0,eff}$ also essentially determines the temperature sensitivity of the line and is related to the number of atoms in the stage of excitation responsible for producing the given line. When η_0 increases, each of the three components of the split line, π, σ_b, and σ_r, is enhanced, becoming deeper and broader. Their degree of overlap also increases in the process. For circular polarization there is one zero crossing of the absorption profile, while for linear polarization there are two, and hence a greater degree of overlap. Figure 3b shows that with increasing η_0 the intensity profile increases continuously, the circular polarization profile varies slowly over some range in η_0 but ultimately begins to decrease, and the linear polarization profile already falls off fairly rapidly for η_0 above 20.

Let us analyze solutions of the radiative transfer equations for the Milne–Eddington atmosphere. Those solutions, which transform the absorption profile into a line profile and are proportional to $\eta_\lambda/(\eta_\lambda + 1)$, can be divided into three parts in the presence of a magnetic field (anomalous dispersion can be neglected to first order),

$$\frac{I_c-I}{I_c} \sim \left(1-\frac{\eta_I+1}{(\eta_I+1)^2-(\eta_Q^2+\eta_V^2)}\right),$$

$$\frac{Q_c-Q}{I_c} \sim \frac{\eta_Q}{(\eta_I+1)^2-(\eta_Q^2+\eta_V^2)},$$

$$\frac{V_c-V}{I_c} \sim \frac{\eta_V}{(\eta_I+1)^2-(\eta_Q^2+\eta_V^2)},$$

where η_I, η_Q, and η_V are determined by the relationship between the splitting components, the Voigt profile, and η_0 (see Ref. 11). As η_0 becomes infinite, the fractions on the right-hand sides of all three equations vanish, i.e., the intensity profile becomes saturated, and the polarization profiles disappear.

A ratio of sensitivities of different Stokes parameters to changes in η_0 will obviously depend on the degree of overlap of the components, determined by the magnetic splitting $v_H = \Delta\lambda_H/\Delta\lambda_D$, and a number of other factors, particularly the value of $\eta_{0,eff}$ itself. For small v_H (and fairly large η_0), the picture does not change qualitatively; on the contrary, with complete splitting of the components, the difference in the dependences of the I, V, and Q profiles on η_0 and a disappears.

The difference in the temperature sensitivities of the different Stokes parameters is thus explained by a combination of temperature variations of the damping constant a and of the ratio η_0 of coefficients of selective and continuous absorption, and by their influence on the resultant I, V, and Q profiles.

The functions that have been found can be useful in selecting lines for the analysis of a two-component umbra, penumbral filaments, and facular points.

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